Simple Slopes and Johnson-Neyman Probing Methods Extended to Two-Condition Within-Subject Designs

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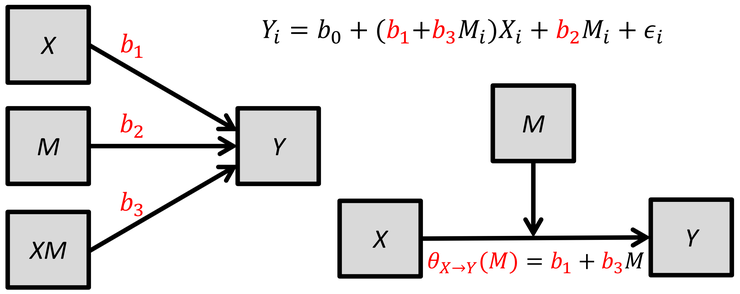
Introduction

Moderation analysis is well-developed and widely-used in *between-subjects*, correlational or experimental designs, and primarily relies on including an **interaction** term in a multiple regression equation. This provides estimates of the *conditional effect* of the focal predictor (*X*) on the outcome (*Y*) conditional on the moderator (*M*).

$$Y_i = b_0 + (b_1 + b_3 M_i)X_i + b_2 M_i + \epsilon_i$$



Two-condition within subjects designs, where each participant is measured in two conditions or at two time points, are very common in psychology. Moderation analysis in these designs examines if the effect of condition (manipulated within participants, e.g. happy story, sad story) on some outcome (measured in each condition; e.g., helping) depends on a moderator (measured once for each person and assumed constant over condition; e.g. empathy).

Methods for Probing

Probing is a method for estimating and conducting inference on conditional effects. It allows the researcher to understand the pattern of conditional effects across the range of the moderator.

Simple slopes (AKA Pick-a-point). In this method researchers select a value of the moderator for which they would like to estimate the conditional effect of *X* on *Y*. An estimate of this effect and it's standard error is calculated and compared to a *t*-distribution with proper degrees of freedom.

$$\frac{\widehat{b_1} + \widehat{b_3}M}{\widehat{var}(\hat{b}_1) + M^2\widehat{var}(\hat{b}_3) + 2M\widehat{cov}(\hat{b}_1, \hat{b}_3)} \sim t_{df}$$

Johnson-Neyman Procedure. This method derives the values of the moderator for which there is a statistically significant effect of *X* on *Y* at a specific level of significance (α). These regions are defined by deriving the boundaries of significance.

$$M = \frac{-\left(2b_3b_1 - 2t^{*2}_{df,\frac{\alpha}{2}}cov(b_1,b_3)\right) \pm \sqrt{\left(2b_1b_3 - 2t^{*2}_{df,\frac{\alpha}{2}}cov(b_1,b_3)\right)^2 - 4\left(b_1^2 - t^{*2}_{df,\frac{\alpha}{2}}var(b_1)\right)\left(b_3^2 - t^{*2}_{df,\frac{\alpha}{2}}var(b_3)\right)}}{2\left(b_3^2 - t^{*2}_{df,\frac{\alpha}{2}}var(b_3)\right)}$$

$t^{*}_{df,\frac{\alpha}{2}}$ is critical *t*-value for a test with *df* degrees of freedom at llevel α

Moderation in Two-Condition Within-Subjects Designs

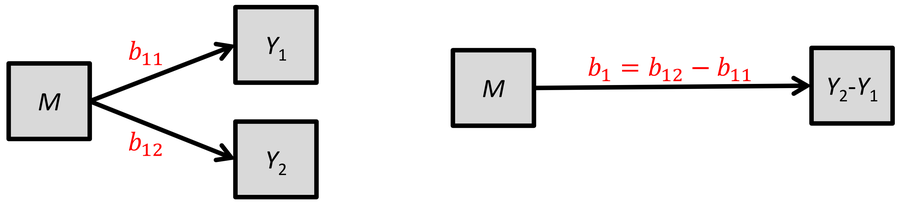
Judd et al. (1996, 2001) showed how to test for an interaction in two-condition within-subjects designs by first setting up models for the outcome in each condition, then taking the difference between them.

$$Y_{i1} = b_{01} + b_{11}M_i + \epsilon_{i1}$$

$$Y_{i2} = b_{02} + b_{12}M_i + \epsilon_{i2}$$

$$\underbrace{Y_{i2} - Y_{i1}}_{Y_{Di}} = \underbrace{b_{02} - b_{01}}_{b_0} + \underbrace{(b_{12} - b_{11})}_{b_1}M_i + \underbrace{(\epsilon_{i2} - \epsilon_{i1})}_{\epsilon_i}$$

The coefficient $\boldsymbol{b_1}$ indicates whether the relationship between *M* and *Y* depends on condition, and whether the relationship between condition and *Y* depends on *M*.



Probing in Two-Condition Within Subjects Designs

Conditional Effects. The conditional effect of condition on *Y* at specific values of *M* is the difference in expected outcomes from each condition at a pre-specified value of the moderator.

$$\theta_{C\rightarrow Y}(M) = b_0 + b_1 M$$

The conditional effect of *M* on *Y* in a specific condition is the relationship between *M* and *Y* estimated in the condition of interest.

$$\theta_{M\rightarrow Y}(C) = b_{1C}$$

Simple slopes (AKA Pick-a-point). The conditional effects can be estimated and tested by taking the ratio of the estimate to it's standard error and comparing to a critical *t*-value.

Testing $\widehat{\boldsymbol{\theta}}_{\boldsymbol{C\rightarrow Y}}(\boldsymbol{M})$

$$\frac{\widehat{b_0} + \widehat{b_1}M}{\widehat{var}(\hat{b}_0) + M^2\widehat{var}(\hat{b}_1) + 2M\widehat{cov}(\hat{b}_0, \hat{b}_1)} \sim t_{df}$$

Testing $\widehat{\boldsymbol{\theta}}_{\boldsymbol{M\rightarrow Y}}(\boldsymbol{C})$

$$\frac{\hat{b}_{1C}}{\widehat{var}(\hat{b}_{1C})} \sim t_{df}$$

Johnson-Neyman Procedure. This method can only be used to probe the effect of condition on the outcome along a continuous moderator. The effect of the moderator cannot be analyzed with this method because condition is dichotomous.

$$M = \frac{-\left(2b_1b_0 - 2t^{*2}_{df,\frac{\alpha}{2}}cov(b_0,b_1)\right) \pm \sqrt{\left(2b_1b_0 - 2t^{*2}_{df,\frac{\alpha}{2}}cov(b_0,b_1)\right)^2 - 4\left(b_0^2 - t^{*2}_{df,\frac{\alpha}{2}}var(b_0)\right)\left(b_1^2 - t^{*2}_{df,\frac{\alpha}{2}}var(b_1)\right)}}{2\left(b_1^2 - t^{*2}_{df,\frac{\alpha}{2}}var(b_1)\right)}$$

$t^{*}_{df,\frac{\alpha}{2}}$ is critical *t*-value for a test with *df* degrees of freedom at level α

SPSS and SAS macro: MEMORE

MEMORE is a macro for SPSS and SAS available at *akmontoya.com* that will estimate and probe a moderation model in a two condition within-subjects design.

Model Specification: After running the syntax file, a simple command can be used to run the analysis.

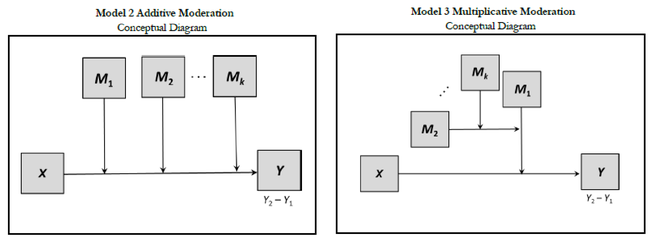
```
MEMORE Y = depA depB /M = mod /Model = 2.
```

This command would estimate and test the interaction between condition and *M*, probe using simple slopes the effect of condition at the mean +/- 1 SD of *M*, and the effect of the *M* in each condition.

Options:

- Johnson-Neyman procedure available for continuous moderators
- Confidence level
- Output code for plots
- Probe at quantiles or specified values
- Multiple moderators (up to 5)

EXAMPLE OUTPUT



Discussion

This research expands methods of **inference for within-subjects moderation to include probing**, which is very popular in between-subjects designs.

Probing allows the researcher to understand the *pattern of effects* along the range of the moderator. Specifically, for what values of the moderator are there significant effects of *X* on *Y*?

MEMORE makes the analysis easy for any researcher to conduct.

This work, in combination with previous work will allow for estimation of **moderated mediation models** in two condition within-subjects designs.

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This poster

Mechanisms and Contingencies Lab

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